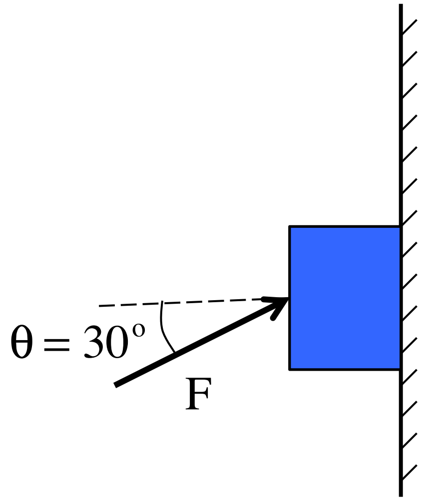
Problem 5.80

A 5.0 kg mass is pushed with constant velocity up a frictional wall whose coefficient of friction $\mu_k = .3$ by a force F directed at an angle of $\theta = 30^o$ as shown in the sketch.



a.) How much work does "F" do?

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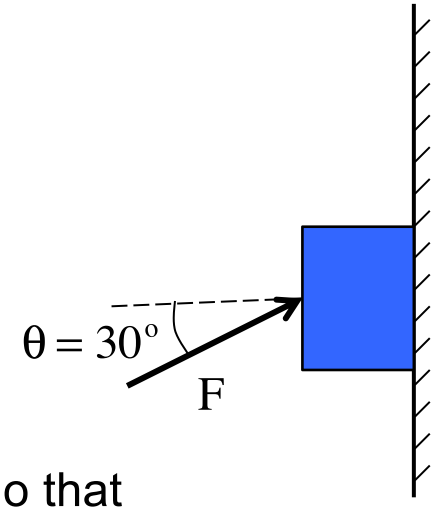
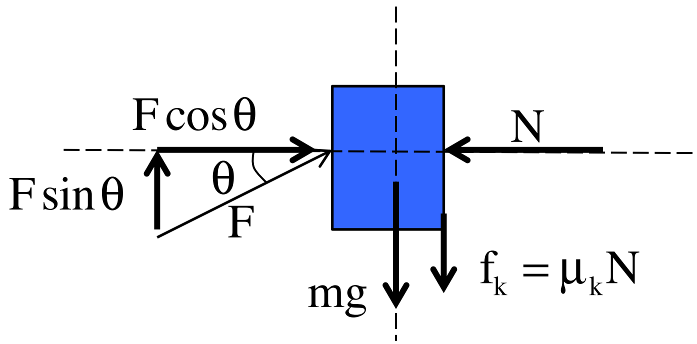


a.) How much work does "F" do?

The temptation is to try to use the dot product, but to do that you need F. If we go that route, we need to use N.S.L. to get F. Starting with a f.b.d. (see to right), we can write:



$$\underline{\sum F_x :}$$

$$-N + F\cos\theta = m\cancel{a_x}^{\,0}$$

$$\Rightarrow \quad N = F\cos\theta$$

and

$$\underline{\sum F_y :}$$

$$F\sin\theta - mg - \mu_k N = m\cancel{a_y}^{\,0}$$

$$F\sin\theta - mg - \mu_k (F\cos\theta) = 0$$

$$\Rightarrow \quad F = \frac{mg}{\sin\theta + \mu_k \cos\theta}$$

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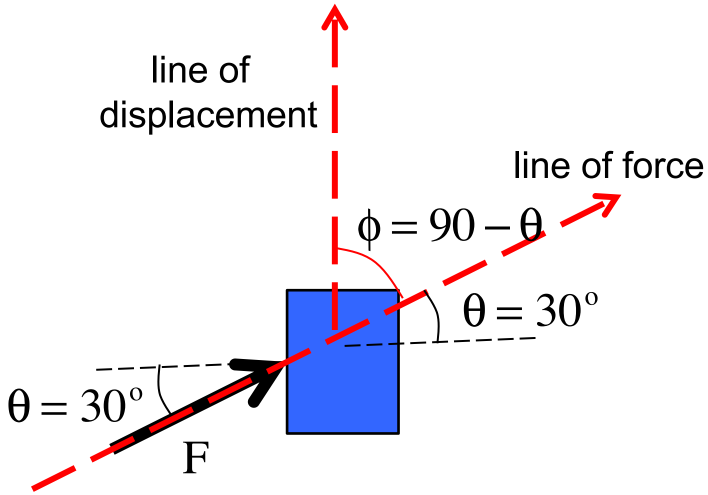
The dot product requires the angle ϕ between the LINE OF F and the LINE OF THE DISPLACEMENT (both shown on the sketch).



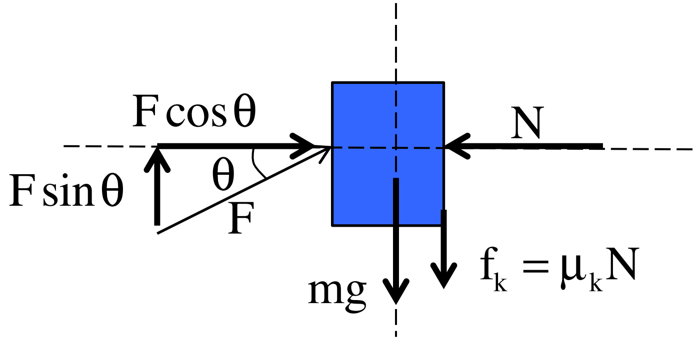
So the dot product becomes:

$$W_F = \vec{F} \bullet \vec{d}$$

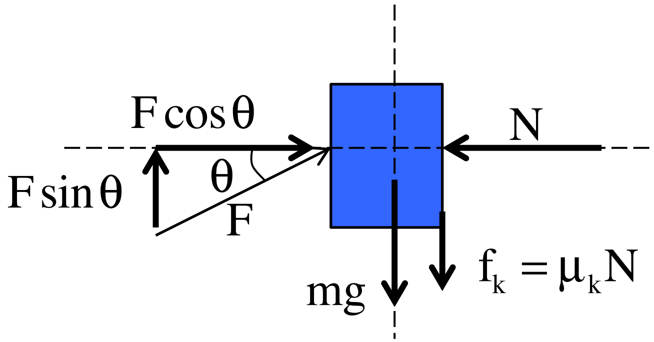
$$= Fd\cos\phi$$

$$= \left(\frac{mg}{\cos\theta + \mu_k \sin\theta}\right) h\cos(90 - \theta)$$

$$= \left(\frac{mg}{\cos\theta + \mu_k \sin\theta}\right) h\sin\theta$$



As an alternative, we could try conservation of energy and see where that takes us. Specifically, remembering that the velocity is constant:



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}mv^2 + 0 + \left[W_f + W_F\right] = \frac{1}{2}mv^2 + mgh$$

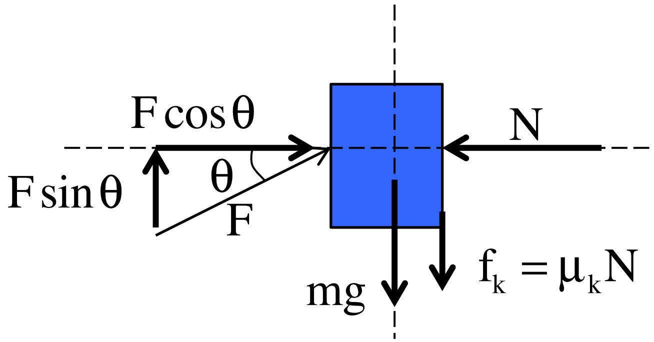
$$\Rightarrow \left[W_f + W_F\right] = mgh$$

Noting that the normal force is $F\cos\theta$ (see sketch), the work due to friction is:

$$\begin{aligned} W_f &= \vec{f} \bullet \vec{d} \\ &= fh\left(\cos 180^\circ\right) \quad [\cos 180^\circ = -1] \\ &= -\left(\mu_k N\right)h \end{aligned}$$

$$\Rightarrow W_F = -\left[\mu_k\left(F\cos\theta\right)\right]h$$

So re-writing the conservation of energy relationship we get:



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}mv^2 + \quad 0 \quad + \left[W_f + W_F\right] = \frac{1}{2}mv^2 + \ mgh$$

$$\Rightarrow \quad \left[W_f + W_F\right] = mgh$$

$$\Rightarrow \quad W_F = \ mgh - W_f$$

$$\Rightarrow \quad W_F = \ mgh - \left(-\mu_k F h \cos\theta\right)$$

But we know that $F = \dfrac{mg}{\sin\theta + \mu_k \cos\theta}$, so we can write:

$$W_F = \ mgh - \left(-\mu_k F h \cos\theta\right)$$

$$= mgh - \left(-\mu_k \left[\frac{mg}{\sin\theta + \mu_k \cos\theta}\right] h \cos\theta\right) \qquad W_F = \left(\frac{mg}{\cos\theta + \mu_k \sin\theta}\right) h \sin\theta$$